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# Speed and shape of solitary waves in nonisothermal plasma with warm ions

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**Abstract** : Large amplitude solitary waves are investigated in a collisionless plasma with nonisothermal electrons and finite ion-temperature. The Sagdeev's pseudopotential is determined in terms of $u$, the ion speed and depends on $v$, the velocity of the wave. It is found that there exists a critical value of $u_0(\neq 0)$ the value of $u$ at which $(u')^2 = 0$, beyond which the solitary waves cease to exist. The critical value also depends on $\beta$, the ratio of the free and the trapped electron respectively.

**Keywords** : Solitary wave, nonisothermal plasma, warm ions.

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## 1. Introduction

Ion-acoustic solitary waves have been studied theoretically and experimentally by several authors [1–14] for the last three decades or so. Washimi and Taniuti [2] first studied the propagation of solitary waves in a simple plasma. Ikezi, Tailor and Baker [14] were among them who first experimentally discovered ion-acoustic solitons (IAS) and double layers in plasma. Many authors had investigated the propagation of solitary waves in plasma considering different plasma models. Das and Tagare [3] derived the Korteweg-de Vries (KdV) equation in a plasma containing negative ions. Das and Paul [4] also studied solitary waves in a weakly relativistic plasma. Several authors worked in this field but most of their studies they did not consider the resonant particles that interact strongly with the wave during its evolution. Schamel [7] first made a theoretical study of ion-acoustic waves due to resonant electron in the frame work of KdV or Modified KdV (MKdV) type equations. Since then, some theoretical works have been done in this field. Parks [9] studied the ion-acoustic solitary waves in a plasma with negative ions and nonisothermal electrons. Also trapped electrons have been observed by Montogomery *et el* [8] in the bow shock. Thompson [10] observed the presence of trapped electron while studying the dynamics of ion beam plasma instability. The experimental support for trapped electrons in plasma waves is found by Wong *et al* [11].

A few years ago Malfliet and Wieers [15] reviewed the studies on solitary waves in plasma and found that RPT (Reductive Perturbative Technique) which is based on the assumption of smallness of amplitude can explain only small amplitude solitary waves. But large amplitude solitary waves also exist in nature. Nakamura *et al* [12] found out large amplitude solitary waves in laboratory plasma. So to study large amplitude solitary waves, one has to employ a nonperturbative approach. Sagdeev's [16] pseudopotential method is one such method which has been successfully applied in various cases [17–19] including multi-component and multi-dimensional plasma.

More recently, Johnston and Epstein [20] studied the nonlinear ion-acoustic solitary waves in a cold collisionless plasma by the direct analysis of the field equations.

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They observed that a very small change in the initial condition destroys the oscillatory behaviour of the solitary waves.

In this paper, our aim is to study large amplitude solitary waves in a plasma with warm ions and two different distribution functions for the electrons, one for the trapped and another for the free electrons. Motivation for taking into account the finite ion temperature is as follows. The ion-temperature in the plasmas of Earth's magnetosphere [21], radiation belt [22] , solar flare [23], solar wind [24] *etc.* can reach hundreds of MeV. For such hot plasmas, the ion-temperature will play an appreciable role in the propagation of solitary waves. Now, the electron density is defined from the Vlasov equations consisting of free and trapped electrons

$$n_e(\phi) = k_0\left[e^{\phi}\,erfc\sqrt{\phi} + \beta^{-1/2}e^{\beta\phi}\,erf\sqrt{\beta\phi}\right]\beta \geq 0 \quad (1)$$

$$= k_0\left| e^{\phi}\,erfc\sqrt{\phi} + (\beta)^{-1/2}\sqrt{\frac{2}{\pi}}\,w\left(\sqrt{-\beta\phi}\right)\right|\beta < 0 ,$$

where $k_0$ is some constant and $\beta = \frac{T_{ef}}{T_{et}}$, where $T_{ef}$ and $T_{et}$ are temperature of the free electrons and trapped electrons, respectively, and

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt, erfc(x) = 1 - erf(x), W(x)$$

$$= e^{-x^2}\int_0^x e^{t^2}dt.$$

In this paper, we see the effect of $\beta$ on the speed and shape of solitary waves by a similar analysis of the field equation as is done in Ref. [24]. We also study the role of $\beta$ on the periodic nature of the solitary waves. Recently, Maitra and Roychoudhury [25] studied the dust-acoustic solitary waves using this technique.

The organization of the paper is as follows. In Section 2 basic equations are written. The governing second order ordinary differential equation is derived. Section 3 is devoted to the results and discussion.

## 2. Basic equations

Our analysis is based on the continuity and momentum fluid equations for ions, electrons and Poisson's equation which are given below.

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \quad (2)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\sigma}{n}\frac{\partial p}{\partial x} = -\frac{\partial \phi}{\partial x}, \quad (3)$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + 3p\frac{\partial u}{\partial x} = 0. \quad (4)$$

The above set of equations is closed by the Poisson's equation,

$$\frac{\partial \phi^2}{\partial x^2} = n_e - n, \quad (5)$$

where we take

$$n_e = e^{\phi}\,erfc\sqrt{\phi} + \beta^{-1/2}e^{\beta\phi}\,erf(\beta\phi)^{\frac{1}{2}}. \quad (6)$$

Here, $n$ and $n_e$ are the normalized ion and electron density respectively, $u$ is the flow velocity of the ion normalized by $\left(\frac{\kappa T_e}{mi}\right)^{\frac{1}{2}}$. $p$ denotes the ion pressure normalized by $(n_0\kappa T_i)^{-1}$, $\phi$ is the electrostatic potential normalized by $\frac{\kappa T_{eff}}{e}$ |. Space and time are normalized by Debye length

$\lambda_D = | \frac{\kappa T_{eff}}{4\pi e^2 n_0}$ and $\omega^{-1}$ where ion plasma frequency

$\omega = \frac{m_i}{4\pi e^2 n_0} |^2$, respectively. $\sigma = \left(\frac{T_i}{T_{eff}}\right)$, where $T_i$ is the ion-temperature and $T_{eff} = \frac{T_{ef}T_{et}}{n_{ef}T_{ef} + n_{et}T_{et}}$, $n_{et}$, $n_{ef}$ being the initial densities of the trapped and free electron respectively, and $n_{et}$ + $n_{ef}$ = 1 and $e$ being the electric charge.

In order to investigate the properties of the solitary wave solutions of eqs. (2) to (5), we assume that all dependent variables depend on the single independent variable $\xi = x - vt$, where $v$ is the velocity of the solitary wave and the variable $\xi$ is the special coordinate in the coordinate system moving with the solitary wave velocity. Now eqs. (2)–(5) reduce to

$$-v\frac{dn}{d\xi}+\frac{d(nu)}{d\xi}=0, \tag{7}$$

$$-v\frac{du}{d\xi}+u\frac{du}{d\xi}+\frac{\sigma}{n}\frac{dp}{d\xi}=-\frac{d\phi}{d\xi}, \tag{8}$$

$$\frac{dp}{d\xi}+u\frac{dp}{d\xi}+3p\frac{du}{d\xi}=0, \tag{9}$$

$$\frac{d^2\phi}{d\xi^2}=n_e-n. \tag{10}$$

Integrating eq. (6) and using the boundary conditions $n \to 1$, $u \to 0$ and $p \to 1$, we get

$$\frac{v}{v-u} \tag{11}$$

Similarly from eq. (8), we have

$$p = n^3, \tag{12}$$

and from eq. (7), we get

$$= uv-\frac{1}{2}u^2-\frac{3\sigma}{2}\left(\frac{v}{v-u}\right)^4+\frac{3\sigma}{2} \tag{13}$$

Now using (11), (12) and (13) in (10), we get

$$\frac{d^2u}{d\xi^2} \quad \frac{\partial\psi(u)}{\partial u} \tag{14}$$

where

$$\psi(u)=-\frac{\psi_e(u)+\psi_i(u)}{(v-u)^2\left|1-\frac{3\sigma v^2}{(v-u)^4}\right.} \tag{15}$$

where

$$u)=-\left[\exp\left(\left(uv-u^2/2\right)\left(1-\frac{3\sigma}{(v-u)}\right.\right.\right.$$

$$\left(\sqrt{\left(\left(uv-u^2/2\right)\left(1-\frac{3\sigma}{(v-u)^2}\right)\right)}\right)$$

$$+\frac{1}{\beta\sqrt{(\beta)}}\exp\left(\beta\left(\left(uv-u^2/2\right)\left(1-\frac{3\sigma}{(v-u)^2}\right)\right)\right)$$

$$erf\left(\sqrt{\beta\left(\left(uv-u^2/2\right)\left(1-\frac{3\sigma}{(v-u)^2}\right)\right)}\right)$$

$$+\frac{2}{\beta\sqrt{\pi}}\sqrt{\left(uv-u^2/2\right)\left(1-\frac{3\sigma}{(v-u)^2}\right)}(\beta-1)+1, \tag{16}$$

$$\psi_i(u)= uv+\sigma v^3\left(\frac{1}{v^3}-\frac{1}{(v-u)^3}\right) \tag{17}$$

If we consider only one species of isothermal electron and Bolzmann distribution for the electron density ($n_e = e^{\phi}$) and neglect the ion-temperature ($\sigma = 0$), then eq. (15) reduces to eq. (22) of Ref. [24].

Expanding *erf* and *erfc* functions and neglecting much higher order terms $O(\phi^5)$, eq. (16) can be written as

$$\psi_e(u)=(uv-u^2/2)\left|1-\frac{3\sigma}{(v-u)^2}\right)+\frac{1}{2}$$

$$(uv-u^2/2)\left(1-\frac{3\sigma}{(v-u)^2}\right.$$

$$\frac{8}{15}b_1\left((uv-u^2/2)\left(1-\frac{3\sigma}{(v-u)^2}\right.\right)^{5/2}$$

$$(uv-u^2/2)\left(1-\frac{3\sigma}{(v-u)^2}\right.$$

$$\frac{16}{105}b_2\left((uv-u^2/2)\left(1-\frac{3\sigma}{(v-u)^2}\right.\right)^{7/2}$$

$$+\frac{1}{24}\left((uv-u^2/2)\left(1-\frac{3\sigma}{(v-u)^2}\right.\right. \tag{18}$$

Hence, $\psi(u)$ and $\frac{d^2u}{d\xi^2}$ can be obtained up to $O(\phi^4)$ from eqs. (14), (15), (17) and (18),

where

$$b_1 = \frac{1-\beta}{\sqrt{\pi}}, \tag{19}$$

$$b_2 = \frac{1-\beta^2}{\sqrt{\pi}}, \tag{20}$$

One can also write

$$\psi(u) = \frac{(u')^2}{2}. \tag{21}$$

**Results and discussion**

To find the region of existence of solitary waves, one has to study the nature of the function $\psi(u)$ and $\phi_1(u)$ defined by

$$u'' = \frac{\partial\psi}{\partial u},$$
$$= \phi_1(u). \tag{22}$$

Solitary wave $\psi(u)$ should be positive through out the region. The point at which $\psi(u)$ crosses, the $u$-axis should be the amplitude of the solitary wave. To get the shape of the travelling solitary wave, one has to solve $\phi_1(u) = u''$ numerically with suitable boundary condition. Figure 1 shows the plot of $\psi$ *vs* $u$ for $v = 1.5$. Other parameters are $\sigma = 0.001$, $\sigma = 0.044$. It is seen that the $\psi(u)$ crosses the $u$-axis at $u = u_0 = 0.585863$. Hence in this case, the

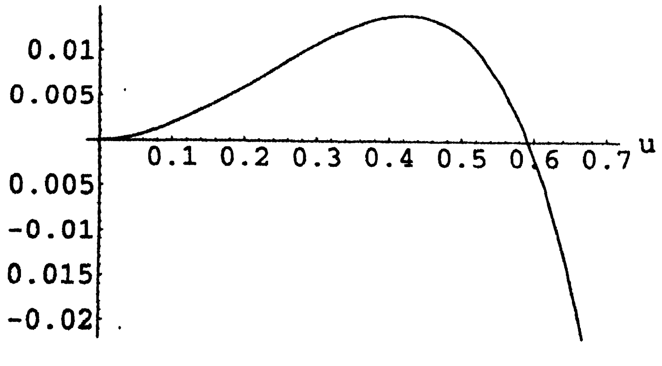


**Figure 1.** The plot of $\psi$ *vs* $u$ with $v = 1.5$, other parameters are same $\sigma = 0.001$ and $\beta = 0.044$.

amplitude of the solitary wave is 0.585863. To get the shape of the solitary wave, we have solved numerically $u'' = \phi_1(u)$ with the initial condition $u_0 = 0.585863$, $u_0' = 0$ and Figure 2(a) depicts the soliton solution $u(\xi)$ plotted against $\xi$. Other parameters are same as that of Figure 1. Hence for this set of parameters, $u_0 = 0.585863$ is the critical value for $u_0$. For $u_0 > 0.585863$, the soliton solution ceases to exist and it is shown in Figure 2(b). In this case, $u_0$ is

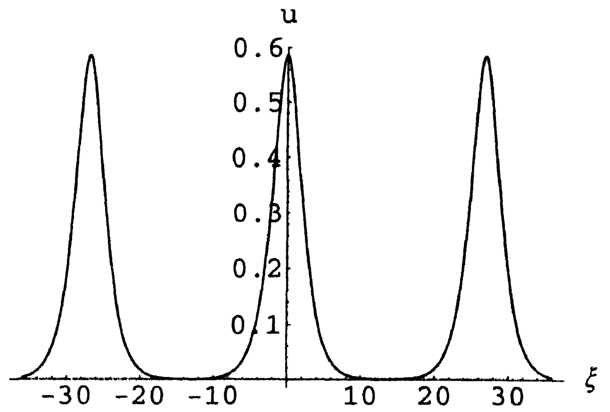


**Figure 2(a).** The soliton of $u(\xi)$ plotted against $\xi$ for $u_0 = 0.585863$ Other parameters are same as those in Figure 1.

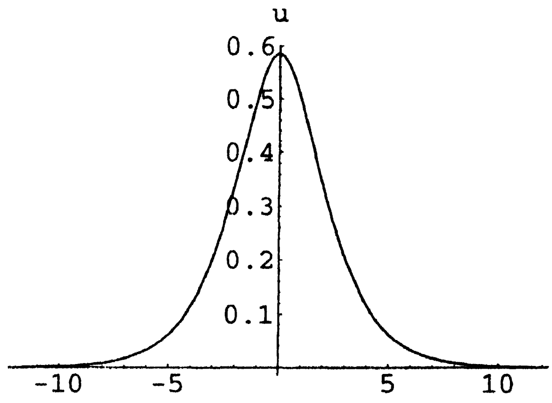


**Figure 2(b).** The soliton of $u(\xi)$ plotted against $\xi$ for $u_0 = 0.585864$. Other parameters are same as those in Figure 1.

taken as 0.585864 (all the other parameters are same as Figure 2(a)). Hence, it is seen that even a small change in $u_0$ can destroy the periodic behaviour of the solitary wave. The divergent part of the wave for the negative value of $u$ cannot be shown because of the presence of square root of $u$ in the differential equation. In Figure 3, $\psi(\xi)$ is plotted against $u$ for different values of $\beta$, *viz.* $\beta = 0.01$, 0.1. Other parameters are $v = 1.5$ and $\sigma = 0.001$. It is seen that the amplitude of the solitary waves increases with the increase of $\beta$. In Figure 4(a), $\psi(u)$ is plotted against $u$ for different values of $\sigma$, *viz.* $\beta = 0.001$. Other parameters are $v = 1.25$, $\beta = 0.04$. Here also, it is seen that the amplitude

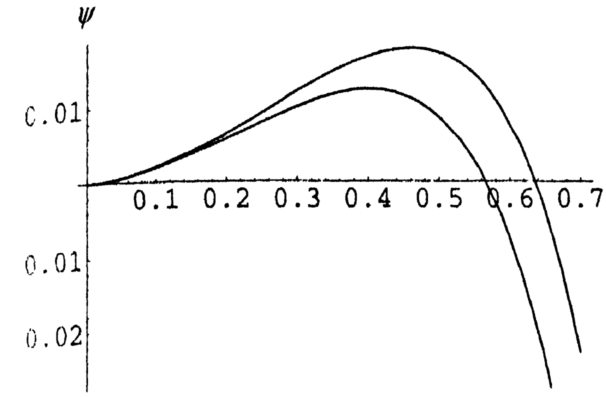


Figure 3. The plot of $\psi$ vs $u$ for different values of $\beta$, viz. $\beta$ = 0.01, 0.1, $\sigma$ = 0.001 and $\upsilon$ = 1.5.

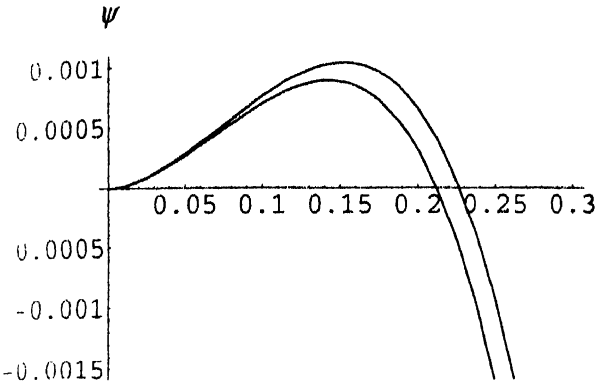


Figure 4(a). The plot of $\psi$ vs $u$ for different values of $\sigma$, viz. $\sigma$ = 0.001, 0.01 $\beta$ = 0.04 and $\upsilon$ = 1.25.

of the solitary wave increases with the increase of ion temperature. In Figure 4(b), soliton solution $u(\xi)$ is plotted against $\xi$; parameters are same as Figure 4(a), Hence, it is seen that $\beta$ and $\sigma$ both play significant role in forming and breaking of solitary waves in plasma.

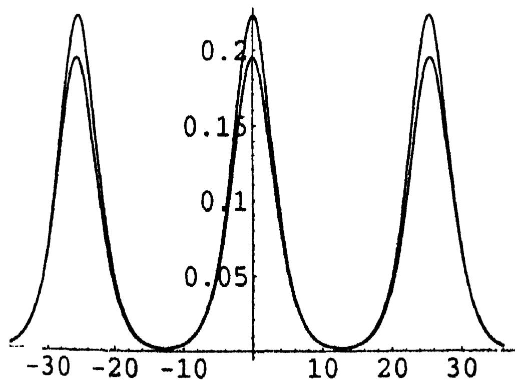


Figure 4(b). The plot of $u$ vs $\xi$ for different values of $\sigma$, viz. $\sigma$ = 0.001, 0.01. $\beta$ = 0.04 and $\upsilon$ = 1.25.

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